Outline

- Clock and Global States
  - Global States
  - Determining Consistent Global States
- Q&A
Global States

- **Prefix of Pi’s History & Global History**
  \[ h_i^k = \langle e_i^j \mid j = 1, ..., k \rangle, \quad H = \bigcup_{i=1}^{N} h_i \]

- **Cut & Frontier**
  \[ C = \bigcup_{i=1}^{N} h_i^{C_i}, \quad F = \{ e_i^{C_i} \mid i = 1, ..., N \} \]

- **Global State (Corresponding to C)**
  \[ S = \{ s_i^{C_i} \mid i = 1, ..., N, \ s_i^{C_i} \text{ is } P_i's \text{ state immediately after } e_i^{C_i} \} \]

- **Run:** a Total Ordering in a Global History Consistent with Each Local History

**Set of All Affected Values**
Consistent Cuts & Runs

C Is Consistent If the Following Holds:

\[ \forall e \in C, \ e' \rightarrow e \Rightarrow e' \in C \]

H Is Consistent If the Corresponding C Is Consistent

Consistent Run: a Total Ordering in a Consistent Global History, Consistent with the Happened-Before Relation
**Lattice of Global States**

**Observing Consistent Global States**

\[ S = \{ s_i \mid i = 1, \ldots, N \} \]  
Is Consistent iff \( VC_i(s_i)[i] \geq VC(s_j)[i] \) for \( i, j = 1, \ldots, N \)

- \( S_{ij} \) is in Level \((i+j)\)

\[ S_{ij} = \text{global state after} \ i \text{ events at process 1 and} \ j \text{ events at process 2} \]
Detecting Global Properties

a. Garbage collection

b. Deadlock

c. Termination
Distributed ‘Snapshot’ Algorithm

[Chandy85]

Consistent Global-State Detection

Marker sending rule for process $p_i$
After $p_i$ has recorded its state, for each outgoing channel $c$:
$p_i$ sends one marker message over $c$
(before it sends any other message over $c$).

Marker receiving rule for process $p_i$
On $p_i$’s receipt of a marker message over channel $c$:
if ($p_i$ has not yet recorded its state) it
records its process state now;
records the state of $c$ as the empty set;
turns on recording of messages arriving over other incoming channels;
else
$p_i$ records the state of $c$ as the set of messages it has received over $c$
since it saved its state.
end if

Assumptions
- Reliable,
- Strongly-Connected Components
- Unidirectional Channels & In-Order Message Delivery
Illustration: How the Alg. Works

- **Initial States of the Components**

Buyer

\[ p_1 \]

$1000

(account)

(none)

(widgets)

Seller

\[ p_2 \]

$50

(account)

2000

(widgets)

P_2 \text{ Has Already Received an Order of Five Widgets}
Illustration (Cont’d)

1. Global state $S_0$

2. Global state $S_1$

3. Global state $S_2$

4. Global state $S_3$

- $p_1: <$1000, 0$>$; $p_2: <$50, 1995$>$; $c_1$: <(five widgets)$>$; $c_2: <>$

$S = p_1: <$1000, 0$>$; $p_2: <$50, 1995$>$; $c_1$: <(five widgets)$>$; $c_2: <>$

P2 Has Already Received an Order of Five Widgets

(M = marker message)
Illustration w/ a Diagram

SS: p0: <>; p1: <e11,e12>; p2:<e21>
c01:<>; c02 <>; c10<
m10>; c12 <
c20 <>; c21<

Scheduler

Physical
time

SS: p0: <>; p1: <e11,e12>; p2:<e21>
c01:<>; c02 <>; c10<
m10>; c12 <
c20 <>; c21<
Consistency Proof

States Recorded by the Alg. Are Consistent:

\[ \forall e_j \in C, \ e_i \rightarrow e_j \Rightarrow e_i \in C \]

Show: \( e_i \notin C, \ e_i \rightarrow e_j \Rightarrow e_j \notin C \)

- Assume That \( P_i \) Recorded Its State before \( e_i \)
- Marker Would Have Reached \( P_j \) before the Message for \( e_j \)
- \( P_j \) Would Have Recorded Its State before \( e_j \)
Characteristics of Snapshots

Derivation of “Observed” Run from “Actual” Run

\[ \text{"Actual" Run: } < e_i^k \mid i = 1, \ldots, N > = < e_j > \]

\[ \text{"Permutated" Run: } < \ldots, e^{R-1}_i, e^R, \ldots > \]

- A Non-Observed Event May Occur before an Observed Event in the “Actual” Run
- If a Non-Observed Event Precedes an Observed Event (Next to it) in the “Actual” Run, the Events Can Be Swapped Preserving Consistency
Global State Predicates

Functions That Map Global States to True or False

- **Stable**: Once True, It Remains True
  - E.g., deadlock or termination

- **Unstable**: Not Stable
  - Possibly True: True At Some Point
    - E.g., snapshot by the ‘Snapshot’ Algorithm
  - Definitely True: True in All Cases