Distributed Information Processing

2nd Lecture

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Outline

- Clock
  - Logical Clocks & Ordering of Events
  - Vector Clocks
- Q&A
Partial Ordering of Events

Assumptions

- System Consists of N Processes:
  \[ p_i \quad (i = 1, \ldots, N) \]

- System Is Composed of a Collection of Events

  - Examples of events
    - Execution of a subprogram
    - Execution of a single machine instruction
    - Sending or receiving a message

\[
\text{history}(p_i) = h_i = \left\langle e_i^k \right\vert k = 1, \ldots \right\rangle \quad (i = 1, \ldots, N)
\]
Partial Ordering of Events
[Lamport78]

"Happened Before" Relation (→)

(1) \( k < l \) \( \Rightarrow \) (if) \( e_i^k \rightarrow e_i^l \) \( (i = 1, \ldots, N) \)

(2) \( e_i = \text{send}(m), e_j = \text{receive}(m) \) for Message \( m \)

\[ \Rightarrow \quad e_i \rightarrow e_j \]

(3) \( e \rightarrow e' \land e' \rightarrow e'' \) \( \Rightarrow \quad e \rightarrow e'' \)

* \( e \nrightarrow e \)

Concurrent Relation (||)

\( e \nrightarrow e^* \land e^* \nrightarrow e \) \( \Leftrightarrow \quad e || e^* \)
Logical Clocks [Lamport78]

- Ways of Assigning Numbers to Events
  
  $LC_i(e)$, where $e$ is an event in process $p_i$

- Clock (Correctness) Condition

  $e \rightarrow e^* \Rightarrow LC(e) < LC(e^*)$

  Satisfied if the following two conditions hold:

  1. $k < l \Rightarrow LC(e^k_i) < LC(e^l_i)$ (for $i = 1, \ldots, N$)
  2. $e_i = send(m)$, $e_j = receive(m)$ for Message $m$

  $\Rightarrow LC(e_i) < LC(e_j)$
Logical Clocks (Cont'd) \cite{Lamport78}

**Implementation Rules**

1. \( LC(e_i^{k+1}) = LC(e_i^k) + 1 \) for Successive Events \( e_i^k, e_i^{k+1} \) \((i = 1, \ldots, N)\)

2. \( e_i = \text{send}(m), e_j^l = \text{receive}(m) \) for Message \( m \)

\[ \Rightarrow t = LC(e_i) \in m, \quad LC(e_j^l) = \max\{ LC(e_j^{l-1}), t \} + 1 \]
Ordering the Events Totally

[Lamport78]

**Total Ordering** \( (\Rightarrow_{t.o.}) \)

\[
e \Rightarrow_{t.o.} e^* \iff \text{LC}_i(e) < \text{LC}_j(e^*) \lor \{\text{LC}_i(e) = \text{LC}_j(e^*) \land i < j\}
\]

\[
e \rightarrow e^* \implies e \Rightarrow_{t.o.} e^*
\]

- **Use Case: Mutual Exclusion Problem**
  - Processes sharing a single resource
    - A process using it must release it before the following use
    - The processes must use it in their request order
    - If it is eventually released, every request is eventually granted
Ordering the Events Totally (Cont’d) [Lamport 78]

Nontrivial Scheduling Problem

The Requests Cannot Be Granted in the Order They Are Received

Scheduler

This Earlier Request Is Received Later

P0

P1

P2

Physical time
Ordering the Events Totally (Cont’d)

Nontrivial Scheduling Problem

Assumptions for an Algorithm

- In-order message delivery
- Each process’ request queue
  - Initially containing a request with T0:P0

Algorithm

- Pi sends a Tm:Pi Req to all others and puts it in the queue
  - Each receiver puts it in the queue and sends a timestamped Ack to the requestor
- When releasing the resource, Pi removes the Req and sends a timestamped Rel to all others
  - Each receiver removes Pi’s Req from its queue
- The Tm:Pi Req is granted if it is the first in ⇒, and Pi has received a message from all others timestamped later than Tm

(1) A process using it must release it before the following use
(2) The processes must use it in their request order
(3) If it is eventually released, every request is eventually granted

Smallest Time Value:
Scheduler ID
Send Time
Vector Clocks

Shortcomings of Lamport’s Logical Clock

\[ LC_i(e) < LC_j(e^*) \quad \Rightarrow \quad e \rightarrow e^* \]
Vector Clocks (Cont’d)

- Implementation Rules

1. \( VC_i[j] = 0 \) (\( i, j = 1, \ldots, N \))

2. \( VC_i(e_i^{k+1})[i] = VC_i(e_i^k)[i] + 1 \) for Successive Events \( e_i^k, e_i^{k+1} \) (\( i = 1, \ldots, N \))

3. \( e_i = send(m), e_j^l = receive(m) \) for Message \( m \)

\[ t = VC_i(e_i) \in m, VC_j(e_j^l) = \max\{ VC_j(e_j^{l-1}), t \} + j \]

Increment the j-th Elt
Vector Clocks (Cont’d)

**Vector-Timestamp Comparisons**

- \( VC = VC^* \iff VC[j] = VC^*[j] \) \((j = 1, ..., N)\)
- \( VC \leq VC^* \iff VC[j] \leq VC^*[j] \) \((j = 1, ..., N)\)
- \( VC < VC^* \iff VC \leq VC^* \land VC \neq VC^* \)

**Properties**

- \( e \rightarrow e^* \iff VC(e) < VC(e^*) \)

**Hints for the Proof**

- \( e \parallel e^* \Rightarrow \neg\{VC(e) \leq VC(e^*) \lor VC(e) \geq VC(e^*)\} \)
- \((Note \ That \ VC_i[j] \leq VC_j[j]\))
Vector Clocks

Properties

\[ e_i \rightarrow e_j \text{ iff } \text{VC}_i[i] \leq \text{VC}_j[i], \text{ where } i \neq j \]

Hint for the Proof

(Note That \( \text{VC}_i[j] \leq \text{VC}_j[j] \))

\[ \exists k(\neq j) \text{ s.t. } \text{VC}_i[k] < \text{VC}_j[k] \]

\[ \Rightarrow \exists e_k \text{ s.t. } (e_k \leftrightarrow e_i) \land (e_k \rightarrow e_j) \]

Simple Clock Condition

Weak Gap Detection
Vector Clocks (Cont’d)

Properties

∃ k (≠ j) s.t. VC_i (e_i)[k] < VC_j (e_j)[k]
⇒ ∃ e_k (≠ e_i) s.t. (e_k → e_i) ∧ (e_k → e_j)

Suppose That i = k,
∃ k (≠ j) s.t. VC_k (e_k^*)[k] < VC_j (e_j)[k]
⇒ ∃ e_k (≠ e_k^*) s.t. (e_k → e_k^*) ∧ (e_k → e_j)
iff ∃ e_k (≠ e_k^*) s.t. e_k^* → e_k → e_j

∃ e_k (≠ e_k^* ∧ k ≠ j) s.t. e_k^* → e_k → e_j
⇒ VC_k (e_k^*)[k] < VC_j (e_j)[k]
Properties

\( \forall (k \neq j), \ VC_k (e_k *)[k] \geq VC_j (e_j)[k] \)

\( \Rightarrow \neg \{ \exists e_k (\neq e_k *) \} \text{ s.t. } e_k^* \rightarrow e_k \rightarrow e_j \)

- If the Monitoring Process Keeps the Value of the \( k \)-th Elt of VC from \( PK \), It Can Decide Whether There Is No Event That Happened before a Given Event

Contraposition